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**(Non)-singular brane-world cosmology induced by
quantum effects in d5 dilatonic gravity**SHIN'ICHI NOJIRI¹, OCTAVIO OBREGON^{♠2},and SERGEI D. ODINTSOV^{♠3},*Department of Applied Physics**National Defence Academy, Hashirimizu Yokosuka 239, JAPAN*[♠] *Instituto de Fisica de la Universidad de Guanajuato**Apdo.Postal E-143, 37150 Leon, Gto., MEXICO***ABSTRACT**

5d dilatonic gravity (bosonic sector of gauged supergravity) with non-trivial bulk potential and with surface terms (boundary cosmological constant and trace anomaly induced effective action for brane quantum matter) is considered. For constant bulk potential and maximally SUSY Yang-Mills theory (CFT living on the brane) the inflationary brane-world is constructed. The bulk is singular asymptotically AdS space with non-constant dilaton and dilatonic de Sitter or hyperbolic brane is induced by quantum matter effects. At the same time, dilaton on the brane is determined dynamically. This all is natural realization of warped compactification in AdS/CFT correspondence. For fine-tuned toy example of non-constant bulk potential we found the non-singular dilatonic brane-world where bulk again represents asymptotically AdS space and de Sitter brane (inflationary phase of observable Universe) is induced exclusively by quantum effects. The radius of the brane and dilaton are determined dynamically. The analytically solvable example of exponential bulk potential leading to singular asymptotically AdS dilatonic bulk space with de Sitter (or hyperbolic) brane is also presented. In all cases under discussion the gravity on the brane is trapped via Randall-Sundrum scenario. It is shown that qualitatively the same types of brane-worlds occur when quantum brane matter is described by N dilaton coupled spinors.

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1 Introduction

After the discovery that gravity on the brane may be localized [1] there was renewed interest in the studies of higher-dimensional (brane-world) theories. In particular, numerous works [2] (and refs. therein) have been devoted to the investigation of cosmology (inflation) of brane-worlds. In refs.[4, 3, 12] it has been suggested the inflationary brane-world scenario realized due to quantum effects of brane matter. Such scenario is based on large N quantum CFT living on the brane [4, 3]. Actually, that corresponds to implementing of RS compactification within the context of renormalization group flow in AdS/CFT set-up. Note that working within large N approximation justifies such approach to brane-world quantum cosmology as then quantum matter loops contribution is essential.

Another important aspect of brane-world Universe with localized gravity is related with the possibility to resolve the cosmological constant problem. For example, it has been shown in ref.[5] that in the presence of bulk scalar (dilaton) one can find static solutions of equations of motion where the bulk dilatonic potential vanishes. Such self-tuning mechanism has been further studied in ref.[8]. Unfortunately, it is usual that such solutions which localize gravity have a naked space-time singularity. (The presence of non-trivial dilatonic potential makes the situation even more complicated [16]). General properties of the self-tuning domain wall solutions of 5d gravity-scalar system with various potentials and brane couplings have been discussed in ref.[17]. It has been shown there that for some specific potential the resolution of singularities (when potential and brane coupling are fine-tuned) may be achieved. The bulk spacetime is asymptotically AdS and gravity localization may occur without having singularities! However, in the studies in this direction the discussion has been done so far mainly for solutions with flat 4d domain walls (flat branes). (The explicit, non-singular example of ref.[17] corresponds to such flat brane configuration). The reason is that in this case the second-order equations of motion may be reduced to first-order form for an arbitrary dilatonic potential, see ref.[9] for explicit examples. In the case when the branes are not flat this procedure does not work directly, generally speaking.⁴

⁴Note that brane-world theory may be often understood as completely 4d two measure theory[6]. Such models with exponential potentials which appear due to scale invariance have been intensively studied in refs.[7](inflation and role of vacuum effects).

The purpose of the present work is to investigate the role of quantum matter living on the brane in the study of brane-world cosmology in 5d AdS dilatonic gravity with non-trivial dilatonic potential (bosonic sector of the corresponding gauged supergravity). We are mainly interested in the situation when the boundary of 5d AdS space represents a 4d constant curvature space whose creation (as is shown) is possible only due to quantum effects of brane matter. Thus, the possibility of dilatonic brane-world inflation induced by quantum effects is proved. In different versions of such scenario discussed here the dynamical determination of dilaton occurs as well.

The paper is organized as follows. In the next section we investigate dilatonic brane-world inflation induced by quantum effects (using anomaly induced effective action) in the situation with constant bulk potential. The bulk space represents (singular) asymptotically AdS background with non-trivial dilaton. The brane matter quantum effects (maximally SUSY Yang-Mills theory is considered as brane CFT) help to create de Sitter or AdS space on the brane. Hence, the quantum realization of brane-world inflation is possible in the presence of the dilaton which is determined dynamically in the bulk as well as on the brane. Note that an analytical treatment is done in this section. Section 3 is devoted to extension of results of previous consideration for non-constant bulk potentials. One solvable example of bulk equations of motion for exponential potential is given. In this case de Sitter (or hyperbolic) brane with small radius occurs for SUSY Yang-Mills theory. It is also interesting that without quantum corrections (W vanishes) the dilatonic hyperbolic brane is still possible.

The conditions to get non-singular, asymptotically AdS dilatonic space-time (when 4d gravity is trapped) are discussed. An example of a toy dilatonic potential is presented. It is shown (in some approximation) that due to quantum effects the brane represents de Sitter space and localization of gravity occurs. Hence, the role of such (fine-tuned) dilatonic potential is to make weaker (or to avoid completely) the singularity which appears for the AdS bulk solution of section 2. Dilaton is still determined dynamically. In section 4 we show that all the above picture may be well realized in the situation when brane matter is not exactly conformal invariant matter (dilaton coupled spinors are considered). Similar qualitative results as in previous sections are obtained. Some resume and perspectives are drawn in the Discussion. In the Appendix a short discussion of some equivalence between 5d dilatonic gravity and 4d dilatonic gravity coupled with CFT is done.

2 Dilatonic brane-world inflation induced by quantum effects: Constant bulk potential

We start with Euclidean signature for the action S which is the sum of the Einstein-Hilbert action S_{EH} with kinetic term for dilaton ϕ , the Gibbons-Hawking surface term S_{GH} , the surface counter term S_1 and the trace anomaly induced action W ⁵:

$$S = S_{\text{EH}} + S_{\text{GH}} + 2S_1 + W, \quad (1)$$

$$S_{\text{EH}} = \frac{1}{16\pi G} \int d^5x \sqrt{g_{(5)}} \left(R_{(5)} - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi + \frac{12}{l^2} \right), \quad (2)$$

$$S_{\text{GH}} = \frac{1}{8\pi G} \int d^4x \sqrt{g_{(4)}} \nabla_\mu n^\mu, \quad (3)$$

$$S_1 = -\frac{3}{8\pi G l} \int d^4x \sqrt{g_{(4)}}, \quad (4)$$

$$\begin{aligned} W = & b \int d^4x \sqrt{\tilde{g}} \tilde{F} A \\ & + b' \int d^4x \left\{ A \left[2\Box^2 + \tilde{R}_{\mu\nu} \tilde{\nabla}_\mu \tilde{\nabla}_\nu - \frac{4}{3} \tilde{R} \Box^2 + \frac{2}{3} (\tilde{\nabla}^\mu \tilde{R}) \tilde{\nabla}_\mu \right] A \right. \\ & + \left(\tilde{G} - \frac{2}{3} \Box \tilde{R} \right) A \Big\} \\ & - \frac{1}{12} \left\{ b'' + \frac{2}{3} (b + b') \right\} \int d^4x \left[\tilde{R} - 6\Box A - 6(\tilde{\nabla}_\mu A)(\tilde{\nabla}^\mu A) \right]^2 \\ & + C \int d^4x A \phi \left[\Box^2 + 2\tilde{R}_{\mu\nu} \tilde{\nabla}_\mu \tilde{\nabla}_\nu - \frac{2}{3} \tilde{R} \Box^2 + \frac{1}{3} (\tilde{\nabla}^\mu \tilde{R}) \tilde{\nabla}_\mu \right] \phi. \quad (5) \end{aligned}$$

Here the quantities in the 5 dimensional bulk spacetime are specified by the suffices ₍₅₎ and those in the boundary 4 dimensional spacetime are specified by ₍₄₎. The factor 2 in front of S_1 in (1) is coming from that we have two bulk regions which are connected with each other by the brane. In (3), n^μ is the unit vector normal to the boundary. In (5), one chooses the 4 dimensional boundary metric as

$$g_{(4)\mu\nu} = e^{2A} \tilde{g}_{\mu\nu}, \quad (6)$$

and we specify the quantities given by $\tilde{g}_{\mu\nu}$ by using $\tilde{\cdot}$. G (\tilde{G}) and F (\tilde{F}) are the Gauss-Bonnet invariant and the square of the Weyl tensor, which are

⁵For the introduction to anomaly induced effective action in curved space-time (with torsion), see section 5.5 in [10].

given as

$$\begin{aligned} G &= R^2 - 4R_{ij}R^{ij} + R_{ijkl}R^{ijkl}, \\ F &= \frac{1}{3}R^2 - 2R_{ij}R^{ij} + R_{ijkl}R^{ijkl}, \end{aligned} \quad (7)$$

⁶ In the effective action (5), we now consider the case corresponding to $\mathcal{N} = 4$ $SU(N)$ Yang-Mills theory, where [11]

$$b = -b' = \frac{C}{4} = \frac{N^2 - 1}{4(4\pi)^2}. \quad (8)$$

The dilaton field ϕ which appears from the coupling with extended conformal supergravity is in general complex but we consider the case in which that only the real part of ϕ is non-zero. Adopting AdS/CFT correspondence one can argue that in symmetric phase the quantum brane matter appears due to maximally SUSY Yang-Mills theory as above. Note that there is a kinetic term for the dilaton in the classical bulk action but also there is dilatonic contribution to the anomaly induced effective action W . Here, it appears the difference with the correspondent construction in ref.[12] where there was no dilaton.

In the bulk, the solution of the equations of motion is given in [14], as follows

$$\begin{aligned} ds^2 &= f(y)dy^2 + y \sum_{i,j=0}^{d-1} \hat{g}_{ij}(x^k) dx^i dx^j \\ f &= \frac{d(d-1)}{4y^2\lambda^2 \left(1 + \frac{c^2}{2\lambda^2 y^d} + \frac{kd}{\lambda^2 y}\right)} \\ \phi &= c \int dy \sqrt{\frac{d(d-1)}{4y^{d+2}\lambda^2 \left(1 + \frac{c^2}{2\lambda^2 y^d} + \frac{kd}{\lambda^2 y}\right)}}. \end{aligned} \quad (9)$$

⁶We use the following curvature conventions:

$$\begin{aligned} R &= g^{\mu\nu} R_{\mu\nu} \\ R_{\mu\nu} &= R^\lambda_{\mu\lambda\nu} \\ R^\lambda_{\mu\rho\nu} &= -\Gamma^\lambda_{\mu\rho,\nu} + \Gamma^\lambda_{\mu\nu,\rho} - \Gamma^\eta_{\mu\rho} \Gamma^\lambda_{\nu\eta} + \Gamma^\eta_{\mu\nu} \Gamma^\lambda_{\rho\eta} \\ \Gamma^\eta_{\mu\lambda} &= \frac{1}{2} g^{\eta\nu} (g_{\mu\nu,\lambda} + g_{\lambda\nu,\mu} - g_{\mu\lambda,\nu}). \end{aligned}$$

Here $\lambda^2 = \frac{12}{l^2}$ and \hat{g}_{ij} is the metric of the Einstein manifold, which is defined by $r_{ij} = k\hat{g}_{ij}$, where r_{ij} is the Ricci tensor constructed with \hat{g}_{ij} and k is a constant. We should note that there is a curvature singularity at $y = 0$ [14]. The solution with non-trivial dilaton would presumably correspond to the deformation of the vacuum (which is associated with the dimension 4 operator, say $\text{tr}F^2$) in the dual maximally SUSY Yang-Mills theory.

If one defines a new coordinate z by

$$z = \int dy \sqrt{\frac{d(d-1)}{4y^2\lambda^2 \left(1 + \frac{c^2}{2\lambda^2 y^d} + \frac{kd}{\lambda^2 y}\right)}}, \quad (10)$$

and solves y with respect to z , we obtain the warp factor $e^{2\hat{A}(z,k)} = y(z)$. Here one assumes the metric of 5 dimensional space time as follows:

$$ds^2 = dz^2 + e^{2A(z,\sigma)} \tilde{g}_{\mu\nu} dx^\mu dx^\nu, \quad \tilde{g}_{\mu\nu} dx^\mu dx^\nu \equiv l^2 (d\sigma^2 + d\Omega_3^2). \quad (11)$$

Here $d\Omega_3^2$ corresponds to the metric of 3 dimensional unit sphere. Then for the unit sphere ($k = 3$), we find

$$A(z, \sigma) = \hat{A}(z, k = 3) - \ln \cosh \sigma, \quad (12)$$

for the flat Euclidean space ($k = 0$)

$$A(z, \sigma) = \hat{A}(z, k = 0) + \sigma, \quad (13)$$

and for the unit hyperboloid ($k = -3$)

$$A(z, \sigma) = \hat{A}(z, k = -3) - \ln \sinh \sigma. \quad (14)$$

We now identify A and \tilde{g} in (11) with those in (6). Then we find $\tilde{F} = \tilde{G} = 0$, $\tilde{R} = \frac{6}{l^2}$ etc.

According to the assumption in (11), the actions in (2), (3), (4), and (5) have the following forms:

$$\begin{aligned} S_{\text{EH}} = & \frac{l^4 V_3}{16\pi G} \int dz d\sigma \left\{ \left(-8\partial_z^2 A - 20(\partial_z A)^2 \right) e^{4A} \right. \\ & + \left(-6\partial_\sigma^2 A - 6(\partial_\sigma A)^2 + 6 \right) e^{2A} \\ & \left. - \frac{1}{2} e^{4A} (\partial_z \phi)^2 - \frac{1}{2l^2} e^{2A} (\partial_\sigma \phi)^2 + \frac{12}{l^2} e^{4A} \right\}, \end{aligned} \quad (15)$$

$$S_{\text{GH}} = \frac{3l^4 V_3}{8\pi G} \int d\sigma e^{4A} \partial_z A, \quad (16)$$

$$S_1 = -\frac{3l^3 V_3}{8\pi G} \int d\sigma e^{4A}, \quad (17)$$

$$\begin{aligned} W = V_3 \int d\sigma & \left[b' A \left(2\partial_\sigma^4 A - 8\partial_\sigma^2 A \right) \right. \\ & - 2(b + b') \left(1 - \partial_\sigma^2 A - (\partial_\sigma A)^2 \right)^2 \\ & \left. + C A \phi \left(\partial_\sigma^4 \phi - 4\partial_\sigma^2 \phi \right) \right]. \end{aligned} \quad (18)$$

Here V_3 is the volume or area of the unit 3 sphere:

$$V_3 = 2\pi^2. \quad (19)$$

On the brane at the boundary, one gets the following equations

$$\begin{aligned} 0 = \frac{48l^4}{16\pi G} & \left(\partial_z A - \frac{1}{l} \right) e^{4A} + b' \left(4\partial_\sigma^4 A - 16\partial_\sigma^2 A \right) \\ & - 4(b + b') \left(\partial_\sigma^4 A + 2\partial_\sigma^2 A - 6(\partial_\sigma A)^2 \partial_\sigma^2 A \right) \\ & + 2C \left(\partial_\sigma^4 \phi - 4\partial_\sigma^2 \phi \right), \end{aligned} \quad (20)$$

from the variation over A and

$$0 = -\frac{l^4}{8\pi G} e^{4A} \partial_z \phi + C \left\{ A \left(\partial_\sigma^4 \phi - 4\partial_\sigma^2 \phi \right) + \partial_\sigma^4 (A\phi) - 4\partial_\sigma^2 (A\phi) \right\}, \quad (21)$$

from the variation over ϕ . We should note that the contributions from S_{EH} and S_{GH} are twice from the naive values since we have two bulk regions which are connected with each other by the brane. The equations (20) and (21) do not depend on k , that is, they are correct for any of the sphere, hyperboloid, or flat Euclidean space. The k dependence appears when the bulk solutions are substituted. Substituting the bulk solution given by (9), (10) and (12), (13) or (14) into (20) and (21), one obtains

$$0 = \frac{1}{\pi G l} \left(\sqrt{1 + \frac{kl^2}{3y_0} + \frac{l^2 c^2}{24y_0^4}} - 1 \right) y_0^2 + 8b', \quad (22)$$

$$0 = -\frac{c}{8\pi G} + 6C\phi_0. \quad (23)$$

Here we assume the brane lies at $y = y_0$ and the dilaton takes a constant value there $\phi = \phi_0$:

$$\phi_0 = \frac{c}{48\pi GC} . \quad (24)$$

Note that eq.(22) does not depend on b and C . Eq.(23) determines the value of ϕ_0 . That might be interesting since the vacuum expectation value of the dilaton cannot be determined perturbatively in string theory. Of course, (24) contains the parameter c , which indicates the non-triviality of the dilaton. The parameter c , however, can be determined from (22). Hence, in such scenario one gets a dynamical mechanism to determine of dilaton on the boundary (in our observable world).

The effective tension of the domain wall is given by

$$\sigma_{\text{eff}} = \frac{3}{4\pi G} \partial_y A = \frac{3}{4\pi Gl} \sqrt{1 + \frac{kl^2}{3y_0} + \frac{l^2 c^2}{24y_0^4}} . \quad (25)$$

We should note that the radial (z) component of the geodesic equation for the in the metric (11) is given by $\frac{d^2 x^z}{d\tau^2} + \partial_z A e^{2A} \left(\frac{dx^t}{d\tau}\right)^2 = 0$. Here τ is the proper time and we can normalize $e^{2A} \left(\frac{dx^t}{d\tau}\right)^2 = 1$ and obtain $\frac{d^2 x^z}{d\tau^2} + \partial_z A = 0$. Since the cosmological constant on the brane is given by $\frac{3}{4\pi G}$, σ_{eff} gives the effective mass density: $\frac{3}{4\pi G} \frac{d^2 x^z}{d\tau^2} = -\sigma_{\text{eff}}$.

As in [3], defining the radius R of the brane in the following way

$$R^2 \equiv y_0 , \quad (26)$$

we can rewrite (22) as

$$0 = \frac{1}{\pi Gl} \left(\sqrt{1 + \frac{kl^2}{3R^2} + \frac{l^2 c^2}{24R^8}} - 1 \right) R^4 + 8b' . \quad (27)$$

Especially when the dilaton vanishes ($c = 0$) and the brane is the unit sphere ($k = 3$), the equation (27) reproduces the result of ref.[3] for $\mathcal{N} = 4$ $SU(N)$ super Yang-Mills theory in case of the large N limit where $b' \rightarrow -\frac{N^2}{4(4\pi)^2}$:

$$\frac{R^3}{l^3} \sqrt{1 + \frac{R^2}{l^2}} = \frac{R^4}{l^4} + \frac{GN^2}{8\pi l^3} . \quad (28)$$

Let us define a function $F(R, c)$ as

$$F(R, c) \equiv \frac{1}{\pi G l} \left(\sqrt{1 + \frac{kl^2}{3R^2} + \frac{l^2 c^2}{24R^8}} - 1 \right) R^4, \quad (29)$$

It appears in the r.h.s. in (27).

First we consider the $k > 0$ case. Since

$$\begin{aligned} \frac{\partial (\ln F(R, c))}{\partial R} &= \frac{1}{R} \left(\sqrt{1 + \frac{kl^2}{3R^2} + \frac{l^2 c^2}{24R^8}} - 1 \right)^{-1} \left(\sqrt{1 + \frac{kl^2}{3R^2} + \frac{l^2 c^2}{24R^8}} \right)^{-1} \\ &\times \left(4 + \frac{kl^2}{R^2} + 4 \sqrt{1 + \frac{kl^2}{3R^2} + \frac{l^2 c^2}{24R^8}} \right)^{-1} \\ &\times \left(\frac{8kl^2}{3R^2} + \frac{k^2 l^4}{R^4} - \frac{2l^2 c^2}{3R^8} \right). \end{aligned} \quad (30)$$

$F(R, c)$ has a minimum at $R = R_0$, where R_0 is defined by

$$0 = \frac{8kl^2}{3R_0^2} + \frac{k^2 l^4}{R_0^4} - \frac{2l^2 c^2}{3R_0^8}. \quad (31)$$

When $k > 0$, there is only one solution for R_0 . Therefore $F(R, c)$ in the case of $k > 0$ (sphere case) is a monotonically increasing function of R when $R > R_0$ and a decreasing function when $R < R_0$. Since $F(R, c)$ is clearly a monotonically increasing function of c , we find for $k > 0$ and $b' < 0$ case that R decreases when c increases if $R > R_0$, that is, the non-trivial dilaton makes the radius smaller. Then, since $1/R$ corresponds to the rate of the inflation of the universe, when we Wick-rotate the sphere into the inflationary universe, the large dilaton supports the rapid universe expansion. Hence, we showed that quantum CFT living on the domain wall leads to the creation of inflationary dilatonic 4d de Sitter-brane Universe realized within 5d AdS bulk space.⁷ Of course, such ever expanding inflationary brane-world is understood in a sense of the analytical continuation of 4d sphere to Lorentzian signature. It would be interesting to understand the relation

⁷Such brane-world quantum inflation for the case of constant dilaton has been presented in refs.[4, 3, 12]. In the usual 4d world the anomaly induced inflation has been suggested in ref.[13] (no dilaton) and in ref.[18] when a non-constant dilaton is present.

between such inflationary brane-world and inflation in D-branes, for example, of Hagedorn type [20].

Since one finds

$$F(R_0, c) = \frac{klR_0^2}{4\pi G}, \quad (32)$$

using (29) and (31), Eq.(27) has a solution if

$$\frac{klR_0^2}{4\pi G} \leq -8b'. \quad (33)$$

That puts again some bounds to the dilaton value. When $|c|$ is small, using (31), one obtains

$$R_0^4 \sim \frac{2c^2}{3k^2l^2}, \quad F(R_0, c) \sim \frac{1}{4\pi G} \frac{|c|}{\sqrt{3}}. \quad (34)$$

Therefore Eq.(33) is satisfied for small $|c|$. On the other hand, when c is large, we get

$$R_0^6 \sim \frac{c^2}{4k}, \quad F(R_0, c) \sim \frac{(k|c|)^{\frac{2}{3}}}{4^{\frac{4}{3}}\pi G}. \quad (35)$$

Therefore Eq.(33) is not always satisfied and we have no solution for R in (27) for very large $|c|$. Then the existence of the inflationary Universe gives a restriction on the value of c , which characterizes the behavior of the dilaton.

We now consider the $k < 0$ case. When $c = 0$, there is no solution for R in (27). Let us define another function $G(R, c)$ as follows:

$$G(R, c) \equiv 1 + \frac{l^2c^2}{24R^8} + \frac{kl^2}{3R^2}. \quad (36)$$

Since $G(R, c)$ appears in the root of $F(R, c)$ in (29), $G(R, c)$ must be positive. Then

$$\frac{\partial G(R, c)}{\partial R} = -\frac{l^2c^2}{3R^9} - \frac{2kl^2}{3R^3}, \quad (37)$$

$G(R, c)$ has a minimum

$$1 + \frac{kl^2}{4} \left(-\frac{2k}{c^2} \right)^{\frac{1}{3}}, \quad (38)$$

when

$$R^6 = -\frac{c^2}{2k}. \quad (39)$$

Therefore if

$$c^2 \geq \frac{k^4 l^6}{32} , \quad (40)$$

$F(R, c)$ is real for any positive value of R . Since

$$F(0, c) = \frac{|c|}{\pi G \sqrt{24}} , \quad (41)$$

and when $R \rightarrow \infty$

$$F(R, c) \rightarrow \frac{klR^2}{6\pi G} < 0 , \quad (42)$$

there is a solution R in (27) if

$$\frac{|c|}{\pi G \sqrt{24}} > -8b' . \quad (43)$$

If we Wick-rotate the solution corresponding to hyperboloid, we obtain a 4 dimensional AdS space, whose metric is given by

$$ds_{\text{AdS}_4}^2 = dz^2 + e^{\frac{2z}{R}} \left(-dt^2 + dx^2 + dy^2 \right) . \quad (44)$$

Eq.(43) tells that there is such kind of solution due to the quantum effect if the parameter c characterizing the behavior of the dilaton is large enough. Thus we demonstrated that due to the dilaton presence there is the possibility of quantum creation of a 4d hyperbolic wall Universe. Again, some bounds to the dilaton appear. It is remarkable that hyperbolic brane-world occurs even for usual matter content due to the dilaton. One can compare with the case in ref.[12] where a hyperbolic 4d wall could be realized only for higher derivative conformal scalar.

In summary, in this section for constant bulk potential, we presented the nice realization of quantum creation of 4d de Sitter or 4d hyperbolic brane Universes living in 5d AdS space. The quantum dynamical determination of dilaton value is also remarkable.

3 Non-constant bulk potentials

We now consider the case that the dilaton field ϕ has a non-trivial potential:

$$\frac{12}{l^2} \rightarrow V(\phi) = \frac{12}{l^2} + \Phi(\phi) . \quad (45)$$

The surface counter terms when the dilaton field ϕ has a non-trivial potential are given in [15]:

$$\begin{aligned}
S^{(2)} &= S_1^\phi + S_2^\phi, \\
S_1^\phi &= -\frac{1}{16\pi G} \int d^4 \sqrt{g_{(4)}} \left(\frac{6}{l} + \frac{l}{4} \Phi(\phi) \right), \\
S_2^\phi &= -\frac{1}{16\pi G} \int d^4 \left\{ \sqrt{g_{(4)}} \left(\frac{l}{2} R_{(4)} - \frac{l}{2} \Phi(\phi) \right. \right. \\
&\quad \left. \left. - \frac{l}{4} \nabla_{(4)} \phi \cdot \nabla_{(4)} \phi \right) - \frac{l^2}{8} n^\mu \partial \left(\sqrt{g_{(4)}} \Phi(\phi) \right) \right\}. \quad (46)
\end{aligned}$$

Following the argument in [3], if one replaces $\frac{12}{l^2}$ in (2) and S_1 in (1) with $V(\phi)$ in (45) and S_1^ϕ in (46), we obtain the gravity on the brane induced by S_2^ϕ . Then if we assume the metric in the following form

$$ds^2 = f(y) dy^2 + y \sum_{i,j=0}^3 \hat{g}_{ij}(x^k) dx^i dx^j, \quad (47)$$

as in (9) and ϕ depends only on y : $\phi = \phi(y)$, we obtain the following equations of motion in the bulk:

$$0 = \frac{3}{2y^2} - \frac{2kf}{y} - \frac{1}{4} \left(\frac{d\phi}{dy} \right)^2 - \left(\frac{6}{l^2} + \frac{1}{2} \Phi(\phi) \right) f, \quad (48)$$

$$0 = \frac{d}{dy} \left(\frac{y^2}{\sqrt{f}} \frac{d\phi}{dy} \right) + \Phi'(\phi) y^2 \sqrt{f}. \quad (49)$$

On the other hand, on the brane, we obtain the following equations instead of (20) and (21):

$$\begin{aligned}
0 &= \frac{48l^4}{16\pi G} \left(\partial_z A - \frac{1}{l} - \frac{l}{24} \Phi(\phi) \right) e^{4A} + b' (4\partial_\sigma^4 A - 16\partial_\sigma^2 A) \\
&\quad - 4(b + b') (\partial_\sigma^4 A + 2\partial_\sigma^2 A - 6(\partial_\sigma A)^2 \partial_\sigma^2 A) \\
&\quad + 2C (\partial_\sigma^4 \phi - 4\partial_\sigma^2 \phi), \quad (50)
\end{aligned}$$

$$\begin{aligned}
0 &= -\frac{l^4}{8\pi G} e^{4A} \partial_z \phi - \frac{l^5}{32\pi G} e^{4A} \Phi'(\phi) \\
&\quad + C \left\{ A (\partial_\sigma^4 \phi - 4\partial_\sigma^2 \phi) + \partial_\sigma^4 (A\phi) - 4\partial_\sigma^2 (A\phi) \right\}. \quad (51)
\end{aligned}$$

In (50) and (51), one assumes the form of the metric as in (11) instead of (47) using the change of the coordinate: $dz = \sqrt{f}dy$ and equations similar to (12), (13), and (14) by choosing $l^2 e^{2\hat{A}(z,k)} = y(z)$.

Using (48) and (49), we can delete f from the equations and we obtain an equation that contains only the dilaton field ϕ :

$$0 = \left\{ \frac{5k}{2} - \frac{k}{4}y^2 \left(\frac{d\phi}{dy} \right)^2 + \left(\frac{3}{2}y - \frac{y^3}{6} \left(\frac{d\phi}{dy} \right)^2 \right) \left(\frac{6}{l^2} + \frac{1}{2}\Phi(\phi) \right) \right\} \frac{d\phi}{dy} + \frac{y^2}{2} \left(\frac{2k}{y} + \frac{6}{l^2} + \frac{1}{2}\Phi(\phi) \right) \frac{d^2\phi}{dy^2} + \left(\frac{3}{4} - \frac{y^2}{8} \left(\frac{d\phi}{dy} \right)^2 \right) \Phi'(\phi) . \quad (52)$$

First we consider a solvable case where

$$\frac{6}{l^2} + \frac{1}{2}\Phi(\phi) = -\frac{2k}{y} . \quad (53)$$

The explicit form, or ϕ dependence, of $\Phi(\phi)$ can be determined after solving the equations of motion. Then since

$$\Phi'(\phi) \frac{d\phi}{dy} = \frac{4k}{y^2}, \quad (54)$$

from (53), Eq.(52) can be rewritten as follows:

$$0 = \left\{ \left(\frac{d\phi}{dy} \right)^2 - \frac{6}{y^2} \right\}^2 . \quad (55)$$

Note that the k -dependence disappears in (55). The solution of (55) is trivially given by

$$\phi = \pm \sqrt{6} \ln(m^2 y) . \quad (56)$$

Here m^2 is a constant of the integration.⁸ Then from (53) and (54), we can find the explicit form of $\Phi(\phi)$:

$$\Phi(\phi) = -\frac{12}{l^2} - 4km^2 e^{\mp \frac{\phi}{\sqrt{6}}} . \quad (57)$$

⁸It is interesting that from AdS/CFT point of view the exponent of above dilaton corresponds to running gauge coupling which has a power behavior in terms of the energy parameter y . This gauge coupling corresponds to a boundary QFT with (broken) supersymmetry.

Note that exponential potentials of the above type often appear as the result of spherical reduction in M-theory or string theory, see discussion in ref.[19]. One can also find that Eq.(48) is trivially satisfied. Integrating (49), we obtain

$$f = \frac{1}{-\frac{2ky}{9} + \frac{f_0}{y^2}} . \quad (58)$$

Here f_0 is a constant of the integration and f_0 should be positive in order that f is positive for large y . There is a (curvature) singularity at $y = 0$. One should also note that when $k > 0$, the horizon appears at

$$y^3 = y_0^3 \equiv \frac{9f_0}{2k} , \quad (59)$$

and we find

$$y \leq y_0 . \quad (60)$$

Then since

$$\partial_z A = \frac{1}{2} \partial_z (\ln y) = \frac{1}{2y} \frac{dy}{dz} = \frac{1}{2y\sqrt{f(y)}} , \quad (61)$$

Eqs.(50) and (51) have the following forms:

$$\begin{aligned} 0 &= \frac{1}{\pi G} \left(\frac{1}{2y_0} \sqrt{\frac{f_0}{y_0^2} - \frac{2ky_0}{9}} - \frac{1}{2l} + \frac{kl}{3y_0} \right) y_0^2 + 8b' \\ &= \frac{1}{\pi G} \left(\frac{1}{2R^2} \sqrt{\frac{f_0}{R^4} - \frac{2kR^2}{9}} - \frac{1}{2l} + \frac{kl}{3R^2} \right) R^4 + 8b' , \end{aligned} \quad (62)$$

$$\begin{aligned} 0 &= -\frac{y_0\sqrt{6}}{8\pi G} \sqrt{\frac{f_0}{y_0^2} - \frac{2ky_0}{9}} - \frac{kly_0\sqrt{6}}{2\pi G} + 6C\phi_0 \\ &= -\frac{R^2\sqrt{6}}{8\pi G} \sqrt{\frac{f_0}{R^4} - \frac{2kR^2}{9}} - \frac{kLR^2\sqrt{6}}{48\pi G} + 6C\phi_0 . \end{aligned} \quad (63)$$

Eq.(63) gives a value of the dilaton on the brane:

$$\phi_0 = \frac{1}{C\sqrt{6}} \frac{R^2}{8\pi G} \left(\sqrt{\frac{f_0}{R^4} - \frac{2kR^2}{9}} + \frac{kl}{6} \right) . \quad (64)$$

When $k > 0$, Eq.(62) does not have a solution for large R since there is an upper bound $R \leq \sqrt{y_0}$ coming from (60). Even for $k \leq 0$, there is no solution

for large R in case of $\mathcal{N} = 4$ Yang-Mills theory ($b' < 0$) since Eq.(62) behaves for large R

$$0 \sim -\frac{1}{2l\pi G}R^4 + 8b' . \quad (65)$$

On the other hand, if one assumes R is small, Eq.(62) has the following form:

$$0 = \frac{1}{\pi G} \left(\frac{\sqrt{f_0}}{2R^4} + \frac{kl}{3R^2} \right) R^4 + 8b' + \mathcal{O}(R^2) , \quad (66)$$

which can be solved with respect to R :

$$R^2 = -\frac{3}{kl} \left(\frac{\sqrt{f_0}}{2} + 8\pi Gb' \right) . \quad (67)$$

Then there is a solution for $k < 0$ ($k > 0$) if

$$f_0 > 128\pi^2 G^2 b'^2 \quad (f_0 < 128\pi^2 G^2 b'^2) . \quad (68)$$

Hence, the results are similar to those in the previous section but in the presence of non-trivial bulk potential.

One can also consider the case of no quantum corrections, i.e. W vanishes. Putting $C = b' = 0$, we obtain from (62) and (63)

$$0 = \frac{1}{2R^2} \sqrt{\frac{f_0}{R^4} - \frac{2kR^2}{9}} - \frac{1}{2l} + \frac{kl}{3R^2}, \quad (69)$$

$$0 = \sqrt{\frac{f_0}{R^4} - \frac{2kR^2}{9}} + \frac{kl}{6} . \quad (70)$$

Eq.(70) tells that $k \leq 0$ but by combining (69) and (70), we find $R^2 = \frac{kl^2}{2}$. Then there is not consistent solution.

Note, however, that the quantum equation (65) for R has the solution for conformally invariant higher derivative scalar whose contribution to b' is positive: $b = -8/120(4\pi)^2, b' = 28/360(4\pi)^2$. In a similar way one can analyze other types of dilatonic potentials (numerically or using some perturbative technique) which lead to (singular) 5d AdS space with 4d constant curvature wall(s).

Let us discuss other examples in attempt to construct non-singular brane-world with inflationary brane induced by quantum effects. As the singularity

usually appears at $y = 0$, we investigate the behavior of (52) when $y \sim 0$. Here we only consider the case $k > 0$. First one assumes that there is no singularity. Then ϕ , $\frac{d\phi}{dy}$, and $\frac{d^2\phi}{dy^2}$ would be finite and we can assume

$$\phi \rightarrow \phi_1 \text{ (constant) when } y \rightarrow 0 . \quad (71)$$

It is supposed the spacetime becomes asymptotically AdS, which is presumably the unique choice to avoid the singularity and to localize gravity on the brane [17]. The condition to get asymptotically AdS requires

$$\Phi'(\phi_1) = 0, \quad (72)$$

and one assumes

$$\Phi'(\phi) \sim \beta \phi_2^\alpha \text{ } (\alpha > 0), \quad \phi_2 \equiv \phi - \phi_1 . \quad (73)$$

Then from (52), one gets

$$0 \sim \frac{5k}{2} \frac{d\phi_2}{dy} + ky \frac{d^2\phi_2}{dy^2} + \frac{3}{4} \beta \phi_2^\alpha . \quad (74)$$

If we also assume ϕ_2 behaves as

$$\phi_2 \sim \tilde{b} y^a \text{ } (a > 0) , \quad (75)$$

one obtains

$$\alpha = 1 - \frac{1}{a} \quad (76)$$

$$\beta = -\frac{4k}{3} \tilde{b}^{\frac{1}{a}} a \left(a + \frac{3}{2} \right) . \quad (77)$$

Eq.(76) requires $0 < \alpha < 1$ and/or $a > 1$ and Eq.(77) tells that β cannot vanish and \tilde{b} should be positive, which tells that ϕ increases when $y \sim 0$.

If we assume $\frac{d\phi}{dy} = 0$ at $y = y_1 > 0$ in (52), we obtain

$$0 = \frac{y_1^2}{2} \left(\frac{2k}{y_1} + \frac{6}{l^2} + \frac{1}{2} \Phi(\phi(y_1)) \right) \frac{d^2\phi}{dy^2} + \frac{3}{4} \Phi'(\phi(y_1)) . \quad (78)$$

In case that $V(\phi) = \frac{12}{l^2} + \Phi(\phi) > 0$, $\frac{d^2\phi}{dy^2} > 0$ ($\frac{d^2\phi}{dy^2} < 0$) if $\Phi'(\phi) < 0$ ($\Phi'(\phi) > 0$). Since ϕ increases when $y \sim 0$, ϕ increases monotonically if $V(\phi) > 0$ and $\Phi'(\phi) < 0$.

One also finds that when y is large, Eq.(52) does not depend on k :

$$0 = \left(\frac{3}{2}y - \frac{y^3}{6} \left(\frac{d\phi}{dy} \right)^2 \right) \left(\frac{6}{l^2} + \frac{1}{2}\Phi(\phi) \right) \frac{d\phi}{dy} + \frac{y^2}{2} \left(\frac{6}{l^2} + \frac{1}{2}\Phi(\phi) \right) \frac{d^2\phi}{dy^2} + \left(\frac{3}{4} - \frac{y^2}{8} \left(\frac{d\phi}{dy} \right)^2 \right) \Phi'(\phi) . \quad (79)$$

Let us consider the following example as a toy model:

$$l^2\Phi(\phi) = -\frac{4}{3}\phi^{\frac{3}{2}} + \frac{3}{4}\phi^4 - \frac{1}{8}\phi^8 + \frac{17}{24} . \quad (80)$$

Since

$$l^2\Phi'(\phi) = -2\phi^{\frac{1}{2}} + 3\phi^3 - \phi^7 , \quad (81)$$

by comparing (81) with (75), one finds

$$\phi_1 = 0 , \quad \alpha = \frac{1}{2} , \quad \beta = -2 , \quad a = 2 , \quad \tilde{b} = \frac{1}{196} . \quad (82)$$

Eq.(81) also tells $\Phi'(\phi) = 0$ when $\phi = 0$ or 1 and $\Phi'(\phi) < 0$ when $0 < \phi < 1$. Then if $\phi \rightarrow 0$ when $y \rightarrow 0$, we can naively expect $\phi \rightarrow 1$ when $y \rightarrow +\infty$. This naive expectation can be confirmed by the numerical calculation, which is given in Figs.1 and 2. In Fig.1, the behavior of ϕ when y is small is given and in Fig.2, the behavior of ϕ when y is large is drawn. From Fig.2, one can find that ϕ goes to unity when y is large. Then there is not any (curvature) singularity and the gravity on the brane can be localized. If we assume

$$1 - \phi \sim \eta y^\xi , \quad \xi < 0 , \quad (\eta \text{ and } \xi \text{ are constant}). \quad (83)$$

the numerical calculation in Fig.2 tells

$$\xi = -0.2 , \quad \eta = 1 . \quad (84)$$

Then from (48), we find the behavior of $f(y)$ when y is large:

$$f \sim \frac{l^2}{4y^2} \left\{ 1 - \frac{\xi^2}{6} \eta^2 \left(\frac{y}{l^2} \right)^{2\xi} + \dots \right\} . \quad (85)$$

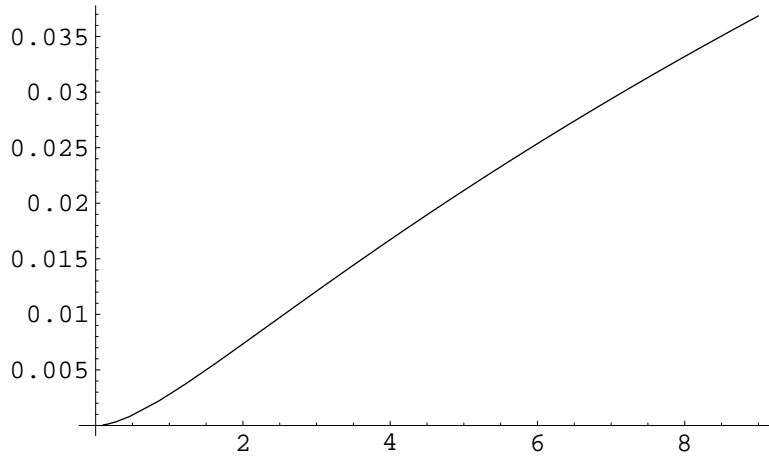


Figure 1: The behavior of ϕ (vertical axis) versus $\frac{y}{l^2}$ (horizontal axis) when y is small.

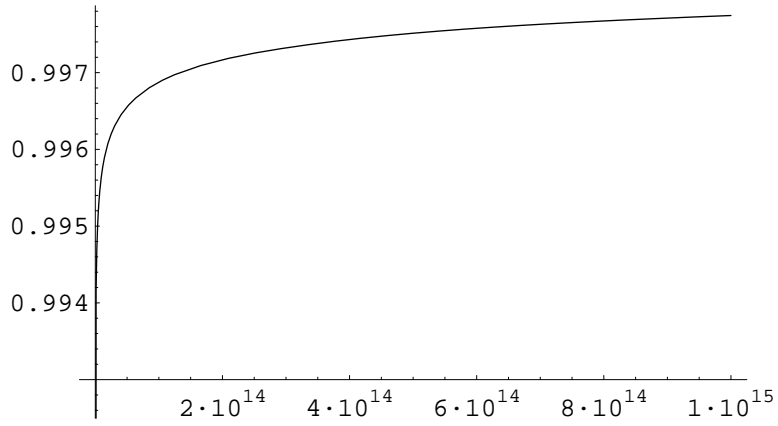


Figure 2: The behavior of ϕ (vertical axis) versus $\frac{y}{l^2}$ (horizontal axis) when y is large.

When y_0 is large, Eqs.(50) and (51) have the following forms:

$$\begin{aligned} 0 &\sim \frac{l^3 \xi^2 \eta^2}{12\pi G} \left(\frac{y_0}{l^2}\right)^{2\xi+2} + 8b' \\ &= \frac{l^3 \xi^2 \eta^2}{12\pi G} \left(\frac{R}{l}\right)^{4\xi+4} + 8b', \end{aligned} \quad (86)$$

$$\begin{aligned} 0 &\sim \frac{l^3 \eta \xi}{4\pi G} \left(\frac{y_0}{l^2}\right)^{\xi+2} + 6C\phi_0 \\ &= \frac{l^3 \eta \xi}{4\pi G} \left(\frac{R}{l}\right)^{2\xi+4} + 6C\phi_0. \end{aligned} \quad (87)$$

Eqs.(86) and (87) can be solved with respect to R and ϕ_0 , respectively:

$$R \sim l \left(-\frac{96\pi G b'}{l^3 \xi^2 \eta^2} \right)^{\frac{1}{4+4\xi}}, \quad (88)$$

$$\phi_0 \sim \left(-\frac{4b'}{C} \right) (\eta \xi)^{-\frac{1}{1+\xi}} \left(-\frac{l^3}{96\pi G b'} \right)^{\frac{\xi}{2+2\xi}}. \quad (89)$$

Since $-b' = \frac{C}{4} = \frac{N^2-1}{4(4\pi)^2}$ from (8) for $\mathcal{N} = 4$ $SU(N)$ Yang-Mills theory and $\frac{G}{l^3} = \frac{\pi}{2N^2}$, Eqs.(88) and (89) tell that

$$R \sim l \left(\frac{3}{4\eta^2 \xi^2} \right)^{\frac{1}{4+4\xi}} = 2.5 \dots, \quad (90)$$

$$\phi_0 \sim (\eta \xi)^{-\frac{1}{1+\xi}} \left(\frac{4}{3} \right)^{\frac{\xi}{2+2\xi}} = 0.13 \dots. \quad (91)$$

Here we also used (84). Since R is not so large, the large y or R approximation converges slowly. Since $0 < \phi_0 < 1$, however, there is no apparent conflict. Eq.(90) shows that the brane does not lie in the asymptotically AdS region when y is large. Anyway it suggests that there is a solution where the brane corresponds to S_4 , which gives the de Sitter space after transition to lorentzian signature.

For comparison, one can consider the classical case where W vanishes. Then (50) and (51) have the following form:

$$0 = \frac{1}{2y\sqrt{f(y)}} - \frac{1}{l} - \frac{l}{24}\Phi(\phi), \quad (92)$$

$$0 = \frac{1}{\sqrt{f(y)}} \frac{d\phi}{dy} - \frac{l}{4} \Phi'(\phi) . \quad (93)$$

Here it is used

$$\partial_z A = \frac{1}{2y\sqrt{f(y)}} , \quad \frac{d}{dz} = \frac{1}{\sqrt{f(y)}} \frac{d}{dy} . \quad (94)$$

Since we have

$$f(y) = \frac{\frac{3}{2y^2} - \frac{1}{4} \left(\frac{d\phi}{dy} \right)^2}{\frac{2k}{y} + \frac{6}{l^2} + \frac{1}{2} \Phi(\phi)} , \quad (95)$$

from (48), one can delete f and $\frac{d\phi}{dy}$ in (92), (93), (95) and obtain

$$\frac{2k}{y_0} = l^2 \left(-\frac{3}{8} \Phi'(\phi) + \frac{1}{96} \Phi^2 \right) . \quad (96)$$

For the potential (80), the l.h.s. in (96) is negative when $1 > \phi > \phi_0 \sim 0.000144 \dots$, which is almost all the allowed region ($1 > \phi > 0$) in the solution for the potential in (80). Therefore there is no classical solution for the $k > 0$ case. Then the brane solution corresponding to 4 dimensional sphere or de Sitter space cannot exist without quantum correction coming from W .

Thus, using fine-tuned dilatonic potential in AdS dilatonic gravity we presented non-singular asymptotically AdS bulk space with de Sitter brane living on the boundary. The dilatonic de Sitter brane is induced by quantum effects of the CFT on the wall. As one can see, gravity trapping occurs. The values of brane radius and of dilaton are dynamically determined.

4 Not exactly conformal brane quantum matter

In this section, we consider the case that the matter on the brane is not the exact CFT like super Yang-Mills theory but some exactly non-conformal theory like QED or QCD. Of course, such a theory is classically a conformally invariant one. As an explicit example in order to be able to apply large N -expansion we suppose that dominant contribution is due to N massless

Majorana spinors coupled with the dilaton, whose action is given by

$$S = \int \sqrt{g_{(4)}} e^{a\phi} \sum_{i=1}^N \bar{\Psi}_i \gamma^\mu D_\mu \Psi_i . \quad (97)$$

The case of minimal spinor coupling corresponds to the choice $a = 0$. Then the trace anomaly induced action W corresponding to (5) has the following form [4]:

$$\begin{aligned} W = & b \int d^4x \sqrt{\tilde{g}} \tilde{F} A_1 \\ & + b' \int d^4x \left\{ A_1 \left[2\Box^2 + \tilde{R}_{\mu\nu} \tilde{\nabla}_\mu \tilde{\nabla}_\nu - \frac{4}{3} \tilde{R} \Box^2 + \frac{2}{3} (\tilde{\nabla}^\mu \tilde{R}) \tilde{\nabla}_\mu \right] A_1 \right. \\ & + \left(\tilde{G} - \frac{2}{3} \Box \tilde{R} \right) A_1 \Big\} \\ & - \frac{1}{12} \left\{ b'' + \frac{2}{3} (b + b') \right\} \int d^4x \left[\tilde{R} - 6\Box A_1 - 6(\tilde{\nabla}_\mu A_1)(\tilde{\nabla}^\mu A_1) \right]^2 . \end{aligned} \quad (98)$$

Here

$$A_1 = A + \frac{a\phi}{3} , \quad (99)$$

and

$$b = \frac{3N}{60(4\pi)^2} , \quad b' = -\frac{11N}{360(4\pi)^2} . \quad (100)$$

We also choose $b'' = 0$ as it may be changed by finite renormalization of classical gravitational action.

First one considers a constant potential ($\Phi(\phi) = 0$). Then the behavior of the solution in the bulk do not change with respect to those in Section 2. On the brane, we obtain the following equations corresponding to (20) and (21):

$$\begin{aligned} 0 = & \frac{48l^4}{16\pi G} \left(\partial_z A - \frac{1}{l} \right) e^{4A} + b' \left(4\partial_\sigma^4 A_1 - 16\partial_\sigma^2 A_1 \right) \\ & - 4(b + b') \left(\partial_\sigma^4 A_1 + 2\partial_\sigma^2 A_1 - 6(\partial_\sigma A_1)^2 \partial_\sigma^2 A_1 \right) , \end{aligned} \quad (101)$$

$$0 = -\frac{l^4}{8\pi G} e^{4A} \partial_z \phi + \frac{4}{3} ab' \left(4\partial_\sigma^4 A_1 - 16\partial_\sigma^2 A_1 \right) . \quad (102)$$

Then one gets

$$0 = \frac{1}{\pi G l} \left\{ \sqrt{1 + \frac{k l^2}{3 R^2} + \frac{l^2 c^2}{24 R^8}} - 1 \right\} R^4 + 8 b', \quad (103)$$

$$0 = -\frac{c}{8\pi G} + 32 a b'. \quad (104)$$

Note that for minimal spinor coupling the second equation does not have a solution. Eq.(103) is identical with (27). Eq.(104) can be solved with respect to c :

$$c = 32 \times 8\pi G a b', \quad (105)$$

but the boundary value ϕ_0 of ϕ becomes a free parameter. Hence, for constant bulk potential there is again the possibility of quantum creation of a 4d de Sitter or a 4d hyperbolic brane living in 5d AdS bulk space. This occurs even for not exactly conformal invariant quantum brane matter. The details of this scenario are similar to those in section 2.

When there is a non-trivial potential corresponding to (53) or (57), Eq.(62) is not changed but Eq.(63) is changed into

$$0 = -\frac{R^2 \sqrt{6}}{8\pi G} \sqrt{\frac{f_0}{R^4} - \frac{2kR^2}{9}} - \frac{k l R^2 \sqrt{6}}{2\pi G} + 32 a b'. \quad (106)$$

For the potential (80), Eq.(86) is not changed again and instead of (87), we obtain

$$0 = \frac{l^3 \eta \xi}{4\pi G l} \left(\frac{R}{l} \right)^{2\xi+4} + 32 a b'. \quad (107)$$

Eqs.(106) and (107) define the parameter a , which characterizes the dilaton coupling in (97). Since the equations for R are identical with (62) and (86), the expression of the radius is not changed. Then for the potential (80), we have

$$R \sim l \left(-\frac{96\pi G b'}{l^3 \xi^2 \eta^2} \right)^{\frac{1}{4+4\xi}}, \quad (108)$$

if R is large. In this case, however, the value of b' in (100) is different from that of (8) for $\mathcal{N} = 4$ $SU(N)$ Yang-Mills theory and we do not know the value for $\frac{l^3}{G}$ (it may be considered as free parameter). Then the value of R itself will be changed from the one in the previous section.

Hence, we have shown that in case of quantum brane matter different from super Yang-Mills theory still there arises (non)-singular brane-worlds for various dilatonic potentials in d5 AdS dilatonic gravity. As in the previous section gravity is trapped. The brane represents a constant curvature space which may be considered as an inflationary phase of our observable Universe.

5 Discussion

In summary, the role of brane quantum matter effects in the realization of de Sitter or AdS dilatonic branes living in d5 (asymptotically) AdS space is carefully investigated. (We are working with d5 dilatonic gravity). The explicit examples of such dilatonic brane-world inflation are presented for constant bulk dilatonic potentials as well as for non-constant bulk potentials. Dilaton gives extra contributions to the effective tension of the domain wall and it may be determined dynamically from bulk/boundary equations of motion. The main part of discussion has dealing with maximally SUSY Yang-Mills theory (exact CFT) living on the brane. However, in section 4 we demonstrated that qualitatively the same results may be obtained when not exactly conformal quantum matter (like classically conformally invariant theory of dilaton coupled spinors) lives on the brane. An explicit example of toy (fine-tuned) dilatonic potential is presented for which the following results are obtained from the bulk/boundary equations of motion: 1. Non-singular asymptotically AdS space is the bulk space. 2. The brane is described by de Sitter space (inflation) induced by brane matter quantum effects. 3. The localization of gravity on the brane occurs. The price to avoid the bulk naked singularity is the fine-tuning of dilatonic potential and dynamical determination (actually, also a kind of fine-tuning) of dilaton and radius of de Sitter brane. Note also that in the same fashion as in ref.[3] one can show that the brane CFT strongly suppresses the metric perturbations (especially, on small scales).

One can easily generalize the results of this work in different directions. For example, taking into account that it is not easy to find new dilatonic bulk solutions like asymptotically AdS space presented in this work one can think about changes in the structure of the boundary manifold. One possibility is in the consideration of a Kantowski-Sachs brane Universe. Another important question is related with the study of cosmological perturbations

around the founded backgrounds and of details of late-time inflation and exit from inflationary phase in brane-world cosmology (eventual decay of de Sitter brane to FRW brane). It would be also interesting to study more examples of dilatonic potentials within the action and brane-world structure under consideration. Clearly, this can be done numerically or using some perturbative expansion of the potential.

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Appendix

AdS₅/CFT₄ correspondence tells us that the effective action W_{CFT} of CFT in 4 dimensions is given by the path integral of the supergravity in 5 dimensional AdS space:

$$\begin{aligned}
e^{-W_{\text{CFT}}} &= \int [dg][d\varphi] e^{-S_{\text{grav}}}, \\
S_{\text{grav}} &= S_{\text{EH}} + S_{\text{GH}} + S_1 + S_2 + \dots, \\
S_{\text{EH}} &= \frac{1}{16\pi G} \int d^5x \sqrt{g_{(5)}} \left(R_{(5)} + \frac{12}{l^2} + \dots \right), \\
S_{\text{GH}} &= \frac{1}{8\pi G} \int d^4x \sqrt{g_{(4)}} \nabla_\mu n^\mu, \\
S_1 &= -\frac{1}{8\pi G l} \int d^4x \sqrt{g_{(4)}} \left(\frac{3}{l} + \dots \right), \\
S_2 &= -\frac{1}{16\pi G l} \int d^4x \sqrt{g_{(4)}} \left(\frac{1}{2} R_{(4)} + \dots \right), \\
&\dots
\end{aligned} \tag{109}$$

Here φ expresses the (matter) fields besides the graviton. S_{EH} corresponds to the Einstein-Hilbert action and S_{GH} to the Gibbons-Hawking surface counter term and n^μ is the unit vector normal to the boundary. S_1, S_2, \dots correspond to the surface counter terms, which cancel the divergences when the boundary in AdS₅ goes to the infinity.

In [3], two 5 dimensional balls $B_5^{(1,2)}$ are glued on the boundary, which is 4 dimensional sphere S_4 . Instead of S_{grav} , if one considers the following

action S

$$S = S_{\text{EH}} + S_{\text{GH}} + 2S_1 = S_{\text{grav}} + S_1 - S_2 - \dots, \quad (110)$$

for two balls, using (109), one gets the following boundary theory in terms of the partition function [3]:

$$\begin{aligned} & \int_{B_5^{(1)} + B_5^{(1)} + S_4} [dg][d\varphi] e^{-S} \\ &= \left(\int_{B_5} [dg][d\varphi] e^{-S_{\text{EH}} - S_{\text{GH}} - S_1} \right)^2 \\ &= e^{2S_2 + \dots} \left(\int_{B_5} [dg][d\varphi] e^{-S_{\text{grav}}} \right)^2 \\ &= e^{-2W_{\text{CFT}} + 2S_2 + \dots} . \end{aligned} \quad (111)$$

Since S_2 can be regarded as the Einstein-Hilbert action on the boundary, which is S_4 in the present case, the gravity on the boundary becomes dynamical. The 4 dimensional gravity is nothing but the gravity localized on the brane in the Randall-Sundrum model [1].

For $\mathcal{N} = 4$ $SU(N)$ Yang-Mills theory, the AdS/CFT dual is given by identifying

$$l = g_{\text{YM}}^{\frac{1}{2}} N^{\frac{1}{4}} l_s, \quad \frac{l^3}{G} = \frac{2N^2}{\pi}. \quad (112)$$

Here g_{YM} is the coupling of the Yang-Mills theory and l_s is the string length. Then (111) tells that the RS model is equivalent to a CFT ($\mathcal{N} = 4$ $SU(N)$ Yang-Mills theory) coupled to 4 dimensional gravity including some correction coming from the higher order counter terms with a Newton constant given by

$$G_4 = G/l. \quad (113)$$

This is an excellent explanation [3] to why gravity is trapped on the brane.

In case that we include the dilaton field (generally with the dilaton potential), the explicit forms of the actions are given by

$$\begin{aligned} S_{\text{EH}}^\phi &= \frac{1}{16\pi G} \int d^5x \sqrt{g_{(5)}} \left(R_{(5)} - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi + \frac{12}{l^2} + \Phi(\phi) \right), \\ S_1^\phi &= -\frac{1}{16\pi G} \int d^4x \sqrt{g_{(4)}} \left(\frac{6}{l} + \frac{l}{4} \Phi(\phi) \right), \\ S_2^\phi &= -\frac{1}{16\pi G} \int d^4x \left\{ \sqrt{g_{(4)}} \left(\frac{l}{2} R_{(4)} - \frac{l}{2} \Phi(\phi) \right) \right. \end{aligned}$$

$$-\frac{l}{4}\nabla_{(4)}\phi\cdot\nabla_{(4)}\phi\Big)-\frac{l^2}{8}n^\mu\partial_\mu\Big(\sqrt{g_{(4)}}\Phi(\phi)\Big)\Big\} . \quad (114)$$

AdS/CFT tells the effective action W of the boundary field theory is given by

$$e^{-W} = \int [dg][d\phi][d\tilde{\varphi}] e^{-S_{\text{EH}}^\phi - S_{\text{GH}} - S_1^\phi - S_2^\phi + \dots} . \quad (115)$$

Here $\tilde{\varphi}$ express the fields besides the graviton and dilaton. Then if we consider the action

$$S = S_{\text{EH}}^\phi + S_{\text{GH}} + 2S_1^\phi, \quad (116)$$

in the two balls, instead of (110), we obtain the following boundary theory given, instead of (111):

$$\begin{aligned} & \int_{B_5^{(1)} + B_5^{(1)} + S_4} [dg][d\phi][d\tilde{\varphi}] e^{-S} \\ &= \left(\int_{B_5} [dg][d\varphi] e^{-S_{\text{EH}}^\phi - S_{\text{GH}} - S_1^\phi} \right)^2 \\ &= e^{-2W + 2S_2^\phi + \dots} . \end{aligned} \quad (117)$$

As S_2^ϕ contains the Einstein action, there appears the dilatonic gravity localized on the brane. We can also choose the trace anomaly induced action as the effective action W .

Then again in this case, by using the identifications in (112) and coupling, (117) tells that the dilatonic RS model is equivalent to a CFT coupled to 4d dilatonic gravity. Such equivalence may be useful in various explicit calculations.

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